Exercise 29

Match the functions f with the plots of their gradient vector fields labeled I–IV. Give reasons for your choices.

$$f(x, y) = x^2 + y^2$$

Solution

Take the gradient of f.

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$= \langle 2x, 2y \rangle$$

This vector field points to the upper-right in quadrant 1, points to the upper-left in quadrant 2, points to the lower-left in quadrant 3, and points to the lower-right in quadrant 4. This matches with plot III.